Lecture 16 QAM Receivers Part 1

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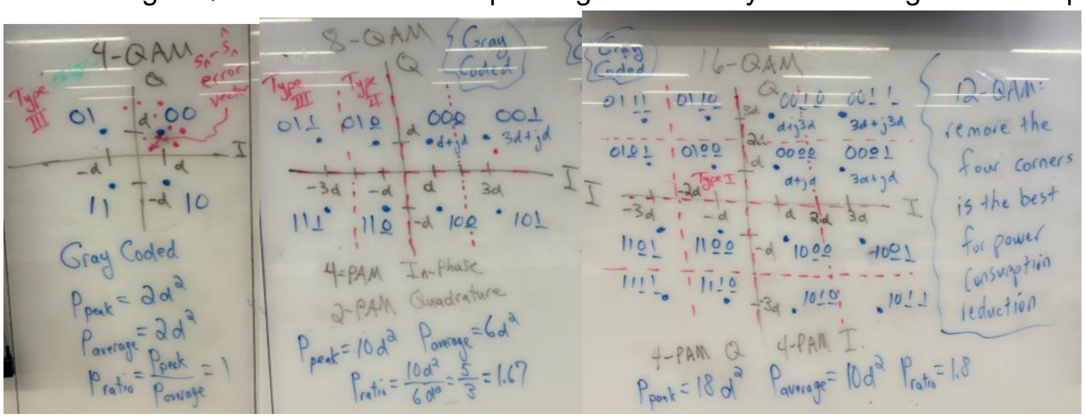
Review of Previous Lecture

- QAM transmission requires phase synchronization at the receiver at much higher accuracy than PAM (PAM can tolerate up to 45 degrees of phase error)
- Marker board pictures below from left to right are
  - 4-QAM: 2-PAM in the in-phase direction, 2-PAM in the quadrature direction
  - 8-QAM: 4-PAM in-phase, 2-PAM quadrature (an alternative would be 2-PAM in-phase, 4-PAM quadrature)
  - 16-QAM: 4-PAM in-phase, 4-PAM quadrature
- Larger QAM constellations require higher accuracy in both magnitude and phase

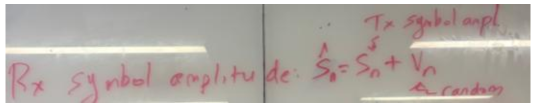


Gray coding

- All three constellations above are Gray Coded
- The baseband receiver's symbol amplitude is the transmitted symbol amplitude plus a random number, where the random number models all the impairments experienced since the constellation mapping in the baseband transmitter
- When there's a symbol error, it's most likely the case that the received symbol amplitude appeared in one the nearest neighbor's constellation regions due to the distribution of the random variable (e.g. Gaussian if the only impairment is additive thermal noise)
- Gray coding takes this into account by encoding the symbol of bits for the nearest neighbor to differ by 1 bit
  - If there is a symbol error, the receiver would be able to detect the bit flip(s) and correct them if error correct coding is being used in the transmitter
  - Error correcting coding takes a block of N message bits and creates a block of N parity bits from message bits and both blocks are transmitted
- When creating the Gray coding, use the first two bits to encode the quadrant and the remaining bits to distinguish the symbols in each quadrant
- Nearest neighbors have symbol amplitudes that are 2d away from each other

Power

- Power at each symbol amplitude: (real part)^2+(imaginary part)^2
- Peak to average power ratio (PAPR) will determine how difficult it is to design our op-amp for the transmit power amplifier
- Larger constellations will have higher PAPRs as shown on the marker board pictures on the first page: 1 for 4-QAM, 1.67 for 8-QAM, and 1.8 for 16-QAM.

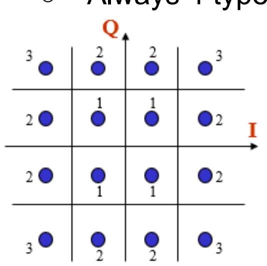


Receiver

- Receiver will get transmitted signal + signal distortion + additive noise and interference
    - Received symbol amplitude can be modeled as the transmitted symbol amplitude + random number (as also mentioned on the previous page)
- Error vector is the distance between the received symbol amplitude and the transmitted symbol amplitude (complex value for QAM and a real value for PAM)
- Want to drive the error vector magnitude to 0 using adaptive algorithms to drive the error vector magnitude squared to 0
    - Cellular LTE standard specifies the relative error (i.e. the absolute value of the error vector / transmitted symbol amplitude times 100%) to be less than 8%
- When deciding which symbol was transmitted,
    - The receiver divides the I-Q into non-overlapping constellation regions whose midpoints are the transmitted symbol amplitudes (a.k.a. constellation points)
    - The receiver selects the transmitted symbol amplitude that has the smallest distance to the received symbol amplitude
        - *Slow Algorithm*: Compute the Euclidean distance from the received symbol amplitude to each transmitter symbol amplitude and the pick the transmitted symbol amplitude that has the smallest distance. Takes 2^*J* Euclidean distance calculations where *J* is the number of bits in a symbol.
        - *Fast Algorithm*: Binary search. Eliminate half of the constellation points at each step through comparing against a threshold. Takes *J* comparisons.

QAM Performance Analysis

- Type 1 constellation region: finite in both dimensions
- Type 2 constellation region: infinite in one dimension and finite in other dimension
- Type 3 constellation region: quarter plane, infinite in 2 directions
    - Always 4 type 3 constellation points to be able to fill out the I-Q plane



- Type 1 constellation, correct detection
    - Received symbol amplitude must be in finite square

- Both inphase and quadrature component of noise must be less than d in absolute value
- Probability of error involves integrating two tails of the random variable PDF in both the inphase and quadrature dimensions
- In-phase and quadrature noise are dependent on temperature, so not actually statistically independent
  - Probability below is an upper bound

$$P_1(c) = P(|v_I(nT_{sym})| < d \;\&\; |v_Q(nT_{sym})| < d)$$

$$= P(|v_I(nT_{sym})| < d)P(|v_Q(nT_{sym})| < d)$$

Assume statistical independence of $v_I(nT_{sym})$ and $v_Q(nT_{sym})$

$$= \Big(1 - \underbrace{P(|v_I(nT_{sym})| > d)}_{2Q(\frac{d}{\sigma}\sqrt{T})}\Big)\Big(1 - \underbrace{P(|v_Q(nT_{sym})| > d)}_{2Q(\frac{d}{\sigma}\sqrt{T})}\Big)$$

$$= \left(1 - 2Q\left(\frac{d}{\sigma}\sqrt{T_{sym}}\right)\right)^2$$

- Type 2 constellation, correct detection
  - One of in-phase or quadrature must be less than d (for the infinite dimension of the constellation region), other will have absolute value less than d (for the finite dimension of the constellation region)
  - Two tails in one dimension, one tail in the other dimension
  - In the first expression immediately after the equal size, P() < d, the +/-d will change based on the exact constellation point

$$P_2(c) = P(v_I(nT_{sym}) < d \;\&\; |v_Q(nT_{sym})| < d)$$

$$= P(v_I(nT_{sym}) < d)P(|v_Q(nT_{sym})| < d)$$

$$= \left(1 - Q\left(\frac{d}{\sigma}\sqrt{T_{sym}}\right)\right)\left(1 - 2Q\left(\frac{d}{\sigma}\sqrt{T_{sym}}\right)\right)$$

- Type 3 constellation, correct detection
  - Both in phase and quadrature are less than d
  - One tail in each dimension
  - +/-d will change based on exact constellation point

$$P_3(c) = P(v_I(nT_{sym}) < d \;\&\; v_Q(nT_{sym}) > -d)$$

$$= P(v_I(nT_{sym}) < d)P(v_Q(nT_{sym}) > -d)$$

$$= \left(1 - Q\left(\frac{d}{\sigma}\sqrt{T_{sym}}\right)\right)^2$$

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- Probability of correct detection
  - Fraction of points in region type * probability of correct detection in that region

- $$P(c)=\frac{4}{16}\left(1-2Q\left(\frac{d}{\sigma}\sqrt{T_{sym}}\right)\right)^2+\frac{4}{16}\left(1-Q\left(\frac{d}{\sigma}\sqrt{T_{sym}}\right)\right)^2 +\frac{8}{16}\left(1-Q\left(\frac{d}{\sigma}\sqrt{T_{sym}}\right)\right)\left(1-2Q\left(\frac{d}{\sigma}\sqrt{T_{sym}}\right)\right) =1-3Q\left(\frac{d}{\sigma}\sqrt{T_{sym}}\right)+\frac{9}{4}Q^2\left(\frac{d}{\sigma}\sqrt{T_{sym}}\right)$$
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- Symbol error probability
  - P(e) = 1-P(c)
  - Might ignore Q^2 term is it is very small; e.g. in the probability of error is say 10^{-7}, then the Q term will be on the order of 10^{-7} and the Q^2 term will be on the order of 10^{-14} which can be ignored

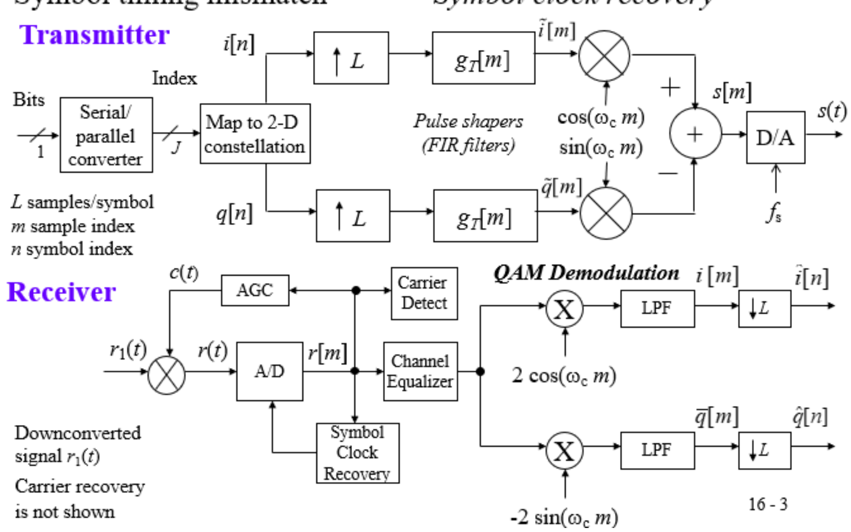
Magnitude/phase importance

- For QAM, phase shifts or amplitude changes can cause receiver to misclassify points
- Both phase and amplitude important for QAM

QAM Receiver

- For each impairment, the receiver has a subsystem to compensate

Fading	*Automatic gain control (AGC)*
Additive noise	*Matched filters*
Linear distortion	*Channel equalizer*
Carrier mismatch	*Carrier recovery*
Symbol timing mismatch	*Symbol clock recovery*



Automatic Gain Control

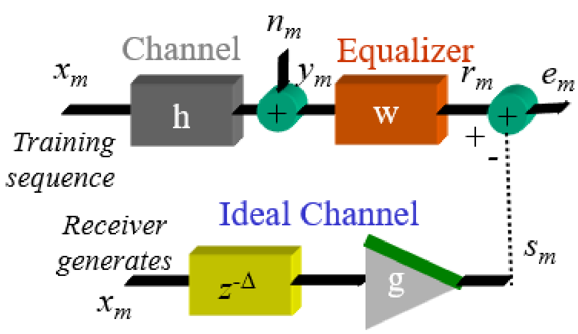
- Scale gain for voltage input to A/D converter

Channel Equalizer

- Mitigate linear distortion in the channel as well as in Tx and Rx analog/RF front ends
- Baseband equivalent channel: cascade of the analog/RF front end of the transmitter, communication channel, and the analog/RF front end of the receiver
- Cascade of LTI distortion in the baseband equivalent channel and equalizer will have a shortened impulse response and near all-pass magnitude response
    - Ideal channel is a fixed delay delta and a fixed gain g

<u>Adaptive equalizer</u>

- Adapt FIR equalizer coefficients when tx sends training sequence to reduce error e_m
- Objective function J is ½ error^2
- Update using steepest descent algorithm with stepsize μ
- Simplest case: two-tap FIR equalizer with filter coefficients 1 and w_1 and adapt w_1



$$e[m] = r[m] - s[m]$$
$$s[m] = g\ x[m - \Delta]$$
$$r[m] = y[m] + w_1\ y[m-1]$$
$$J_{LMS}[m] = \frac{1}{2} e^2[m]$$
$$w_1[m+1] = w_1[m] - \mu \frac{\partial J_{LMS}[m]}{\partial w_1}\bigg|_{w_1 = w_1[m]}$$
$$w_1[m+1] = w_1[m] - \mu\ e[m]\ y[m-1]$$

- General case: *N*-tap FIR equalizer where we adapt all the coefficients:

- **General case: Adapt *N* real coefficients $w_0, w_1, \ldots w_{N-1}$**

$$r[m] = w_0\ y[m] + w_1\ y[m-1] + \cdots + w_{N-1}\ y[m-(N-1)]$$

- **Derive coefficient update equations during training**

Ideal: $s[m] = g\ x[m - \Delta]$ **Error**: $e[m] = r[m] - s[m]$ $J_{LMS}[m] = \frac{1}{2} e^2[m]$

$$w_0[m+1] = w_0[m] - \mu \frac{\partial J_{LMS}[m]}{\partial w_0}\bigg|_{w_0 = w_0[m]} = w_0[m] - \mu\ e[m] \frac{\partial r[m]}{\partial w_0}\bigg|_{w_0 = w_0[m]}$$

$$\underline{\vec{w}[m]} \quad \begin{bmatrix} w_0[m] \\ w_1[m] \\ \vdots \\ w_{N-1}[m] \end{bmatrix}$$

$$w_0[m+1] = w_0[m] - \mu\ e[m]\ y[m]$$
$$w_1[m+1] = w_1[m] - \mu\ e[m]\ y[m-1]$$
$$\vdots$$
$$w_{N-1}[m+1] = w_{N-1}[m] - \mu\ e[m]\ y[m-(N-1)]$$

$$\underline{\vec{y}[m]} \quad \begin{bmatrix} y[m] \\ y[m-1] \\ \vdots \\ y[m-(N-1)] \end{bmatrix}$$

- **Vector form:** $\vec{w}[m+1] = \vec{w}[m] - \mu\ e[m]\ \vec{y}[m]$